PROPAGATION OF A SHOCK PULSE TRAIN IN DENSE MEDIA

É. I. Andriankin, A. I. Malkin, and N. N. Myagkov

UDC 534.222

The investigation of nonstationary shocks in dense media is of interest for many problems of shock physics which occur, for instance, in the analysis of meteor impacts, powerful laser radiation interaction with a substance, shock methods of obtaining new materials, explosions in dense media, etc. [1-5]. In a number of such problems, the propagation of shocks with pressure amplitudes considerably exceeding the shear modulus of the substance but less than the modulus of multilateral compression must be investigated. Hence, to describe the state of the medium in this case, a hydrodynamic approximation [1] is valid, but Burgers equation [6] can be used to analyze the propagation of a shock pulse train with dissipation taken into account. In this paper plane problems are examined. In this case the Burgers equation (BE) is solved exactly for physically interesting boundary conditions and the problem is reduced to extracting the information from the solution obtained.

If a pressure pulse applied to a boundary can be approximated by a simple function of the time, a $\delta$ or step function, say, the analysis of the solution of the BE is not complex [6]. Examination of the problems with more complex boundary conditions is of interest. In particular, evolution of a compressive pulse trains, occurring under successive impacts on a specimen surface, must be investigated for practical applications. The formulation of such a problem is because of the requirement to vary the shape of the pressure pulse applied to the boundary which often occurs in experiments on the shock compression of condensed substances. Pressure pulses, obtained when using impactors, short laser pulses, electronic impacts, and detonation of the layers of condensed high explosives, have a qualitatively similar shape, a steep front, and a small drop-off domain. Hence, in practice it is convenient to realize a change in the compression wave shape by using trains of pressure pulses generated by a pulse laser, say. The selection of the lag time between the laser pulses can yield the possibility of forming a wave with the given parameters in the medium. Moreover, utilization of a laser pulse train will permit diminution of the influence of screening of the surface of the condensed substance by rupture products [2], i.e., optimal conditions for shock formation of comparatively long duration are achieved.

The evolution of weak shock pulse trains is examined below. Estimates of the characteristic distances for merger of the pulses and the amplitudes being shaped during wave merger are obtained. The possibility of increasing the dynamic wave parameters (energy, momentum, etc.) at a given distance from the boundary surface because of the selection of the lag time between the pulses in the train is shown.

1. If the equations of dissipative hydrodynamics closed by an equation of state of Mie–Gruneisen or Tait type, or the ideal gas equation of state are taken as the initial system, then application of the methods of nonlinear wave theory [6] reduce the BE to the form

$$\frac{\partial q}{\partial \eta} - q\frac{\partial q}{\partial \xi} = b\frac{\partial^2 q}{\partial \xi^2}, \quad \eta = \frac{(1+n)\,\omega m}{2K_0}, \tag{1}$$

where $\xi = \omega t - \omega m/K_0$; $b = \frac{\omega}{(1+n)M_0}\left[\alpha + \frac{4}{3}\beta + \varkappa\left(\frac{1}{c_V} - \frac{1}{c_p}\right)\right]$; $q = p/M_0$; $m$ is the Lagrange mass coordinate; $K_0$ and $M_0$, impedance and compression modulus of the medium; $\alpha$, $\beta$, and $\varkappa$, respectively, volume and shear viscosity coefficients and the heat conduction; and, n, exponent in the ideal gas of Tait equation of state, where the $n = 1+\left(\frac{\partial^2 p}{\partial \rho^2}\right)_S \rho_0 \Big/ \left(\frac{\partial p}{\partial \rho}\right)_S$ is a constant for an equation of state of Mie–Gruneisen type. Here p is the pressure, $\rho$ is the density, S is the entropy, and $\omega^{-1}$ is the characteristic time scale from the boundary condition introduced to make (1.1) dimensionless. It must be noted that the BE in the form (1.1) can be obtained also from the non-

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 156-163, May-June, 1982. Original article submitted April 23, 1981.

 

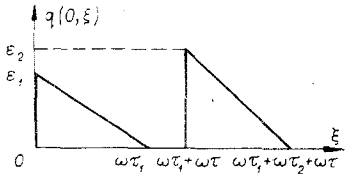


Fig. 1

linear elasticity theory equations [7], the equations describing the motion of certain heterogeneous media [8], etc., where the influence of the specific properties of the medium is felt only in the dependence of the dissipative factor b on the parameters of the medium, in the magnitude of the impedance and compression modulus of the medium.

Let us assume the dimensionless shock pulse amplitude on the boundary to be characterized by a certain small parameter $\varepsilon$. Then the relative influence of the dissipation and nonlinearity on evolution of the wave shape is determined by the Reynolds number $\mathrm{Re} = \varepsilon/4\pi b$. Let us examine two cases of large and moderate Reynolds numbers separately.

2. For large Reynolds numbers the dissipative term in (1.1) can be neglected by eliminating the ambiguity in the wave profile by using the "equal area" rule resulting from (1.1) [6]. On the boundary of the medium let the pressure vary according to the law (Fig. 1)

$$q(0,\xi)=\begin{cases}\varepsilon_1(1-\xi/\omega\tau_1), & \xi\in(0,\omega\tau_1),\\ \varepsilon_2(1-(\xi-\omega\tau_1-\omega\tau)/\omega\tau_2), & \xi\in(\omega\tau_1+\omega\tau,\ \omega\tau_1+\omega\tau_2+\omega\tau),\\ 0, & \xi\notin(0,\ \omega\tau_1)\cup(\omega\tau_1+\omega\tau,\ \omega\tau_1+\omega\tau_2+\omega\tau),\end{cases} \tag{2.1}$$

where $\omega^{-1} = \tau_1 + \tau_2 + \tau$.

Let us consider certain characteristic parameters of the evolution of a train of two shock pulses (2.1) in the limit as $\mathrm{Re} \to \infty$. At a certain distance from the boundary surface the pulses merge completely; the wave profile becomes triangular. The distance from the boundary (in mass) at which the shock discontinuities merge, i.e., the wave emerges into the asymptotic regime [6], is

$$m_{\mathrm{m}}=\frac{2K_0\tau_2}{\varepsilon_2(1+n)}\left[\frac{\varepsilon_1\tau_1}{\varepsilon_2\tau_2}\left(1+\frac{\tau}{\tau_2}\right)^2A^2(1+B)^2-1\right], \tag{2.2}$$

where

$$B=\left(\frac{\varepsilon_2\tau_2+\varepsilon_1\tau_1}{\varepsilon_1\tau_1A^2}\right)^{1/2};\ A^2=1-\frac{\tau_2(\varepsilon_1\tau_2-\varepsilon_2\tau_1)}{\varepsilon_1(\tau+\tau_2)^2}.$$

As follows from (2.2), if the lag time between pulses $\tau$ is much less than the duration of the second pulse, then the merger distance grows linearly with $\tau$; if $\tau \ll \tau_2$, then the growth law is parabolic. At the time of merger, the amplitude and duration of the resultant wave equal

$$q_{\mathrm{m}}=\frac{\varepsilon_2\tau_2}{\tau+\tau_2}\left(1+\frac{1}{B}\right)^{-1}; \tag{2.3}$$

$$\omega^{-1}\Delta\xi_{\mathrm{m}}=\left(\tau_2+\frac{\varepsilon_1}{\varepsilon_2}\tau_1\right)\left(1+\frac{\tau}{\tau_2}\right)\left(1+\frac{1}{B}\right). \tag{2.4}$$

Another important parameter for pulse train investigation is the jump in amplitude during merger of the discontinuities, equal to

$$\Delta q_{\mathrm{m}}=\frac{\varepsilon_2\tau_2^2}{(\tau_2+\tau)\varepsilon_1\tau_1B}\left(1+\frac{1}{B}\right)^{-1}(1+BA^2)^{-1}. \tag{2.5}$$

We determine the quantities $\varepsilon_1\tau_1$ and $\varepsilon_2\tau_2$ proportional to the mechanical momentum transmitted to the medium in the first and second impacts, respectively, in a first approximation. For constant $\varepsilon_1\tau_1$ and $\varepsilon_2\tau_2$ it follows from (2.2) that the merger distance $m_m$ grows as $\tau_1$ increases for $\tau_2$ = const and as $\tau_2$ increases for $\tau_1$ = const. In the second case the growth of $m_m$ is simply related to the diminution of the initial amplitude of the overtaking pulse and, therefore, as the velocity of propagation of the second discontinuity diminishes. In order to explain the growth of $m_m$ with the growth of $\tau_1$, let us note that the propagation velocity of the low discontinuity in the mass of substance equals

$$\frac{dm_1}{dt} = K_0\left[1 + \frac{2\varepsilon_1\tau_1}{(1+n)\sqrt{\tau_1^2 + \frac{2\varepsilon_1\tau_1 m_1}{(1+n)K_0}}}\right]$$

and decreases as $\tau_1$ grows. Hence, the growth of $m_m$ with $\tau_1$ is explained by the increase in the initial distance between discontinuities, which turns out to be more essential than the diminution in the low discontinuity velocity.

In case the durations of both pulses are significantly less than the lag time $\tau$, formulas (2.2)-(2.5) simplify significantly. Formally this simplification corresponds to the passage to the limit $\tau_1$, $\tau_2 \to 0$ for $\varepsilon_1\tau_1$, $\varepsilon_2\tau_2$ = const. For instance, the expressions for the merger distance and amplitude take the form

$$m_m = \frac{2K_0\tau^2}{(1+n)\varepsilon_2\tau_2}\frac{\varepsilon_1\tau_1}{\varepsilon_2\tau_2}\left(\sqrt{\frac{\varepsilon_1\tau_1+\varepsilon_2\tau_2}{\varepsilon_1\tau_1}}+1\right)^2, \quad q_m = \frac{\varepsilon_2\tau_2}{\tau}\left(1+\sqrt{\frac{\varepsilon_1\tau_1}{\varepsilon_1\tau_1+\varepsilon_2\tau_2}}\right)^{-1}.$$

3. Let us now consider the case of finite dissipative factor in (1.1). Since the Hopf-Cole transformation [6] $q = 2b(\ln U)'_\xi$ of the BE results in a linear heat conductivity equation, the solution of (1.1) for an arbitrary boundary condition can be written down at once. As before, let two successive pressure pulses be given on the boundary $\eta = 0$. If the duration of each is considerably less than the lag time $\tau$, then the boundary conditions can be approximated by two $\delta$-functions separated by the time interval $\tau$, i.e.,

$$q(0,\xi) = \varepsilon_1\delta\left(\frac{\xi}{\omega\tau_1}\right) + \varepsilon_2\delta\left(\frac{\xi-\omega\tau}{\omega\tau_2}\right). \tag{3.1}$$

It is convenient to represent the solution of the problem (1.1), (3.1) in the form

$$U = \frac{e^{\Gamma_1+\Gamma_2}-1}{\sqrt{\pi}}\int_{-\infty}^{y} e^{-t^2}dt - \frac{e^{\Gamma_1}\left(e^{\Gamma_2}-1\right)}{\sqrt{\pi}}\int_{0}^{x} e^{-(y-t)^2}dt, \tag{3.2}$$

where

$$\Gamma_1 = \omega\varepsilon_1\tau_1/2b;\ \Gamma_2 = \omega\varepsilon_2\tau_2/2b;\ y = \xi/2\sqrt{b\eta};\ x = \tau\omega/2\sqrt{b\eta}.$$

As $b\eta$ grows, the last term in (3.2) decreases and the solution goes asymptotically into the self-similar regime corresponding to propagation of a single compression pulse. As in the case Re $\to \infty$, we describe the merger of output pulses in the asymptotic regime.

Important characteristics of wave propagation are energy and momentum. In conformity with the quadratic approximation under consideration, momentum in a first approximation $I = \int_{-\infty}^{+\infty} q d\xi$ remains constant, and the total values of the energy and momentum will vary only because of the term $E(\eta) = \int_{-\infty}^{+\infty} q^2 d\xi$. The solution of the problem (1.1), (3.1) is performed numerically, and the energy $J(x) = (\eta/4b^3)^{1/2} E(\eta) = \int_{-\infty}^{+\infty} (U'_y/U)^2$ is represented in Fig. 2 for different $\Gamma_1$ and $\Gamma_2$. Curve 1 corresponds to $\Gamma_1 = 5$, $\Gamma_2 = 5$; 2) $\Gamma_1 = 10$, $\Gamma_2 = 5$; 3) $\Gamma_1 = 5$, $\Gamma_2 = 10$; 4) $\Gamma_1 = 15$, $\Gamma_2 = 5$; 5) $\Gamma_1 = 10$, $\Gamma_2 = 10$; 6) $\Gamma_1 = 5$, $\Gamma_2 = 15$. Each curve in Fig. 2 includes an upper and lower plateau on which J(x) is independent of x. The lower plateau corresponds to independent pulse propagation when interaction is slight, while the energy J(x) is determined by the sum of the pulse energies. Starting with a certain distance from the boundary, the second pulse overtakes the first; they start to interact intensively [J(x) grows here]. For a certain $x_m$ interaction almost ceases (the pulses merge), and for $x < x_m$ the resultant pulse emerges on the asymptotic of the self-similar wave, which corresponds to the solution (3.2) with x = 0, and J is again independent of x. Therefore, the distance $x_m$ of pulse merger can already be estimated from Fig. 2, for example $x_m \approx 0.5$ for curve 4. In order to estimate the characteristic distance of

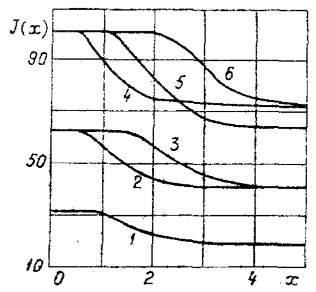

Fig. 2

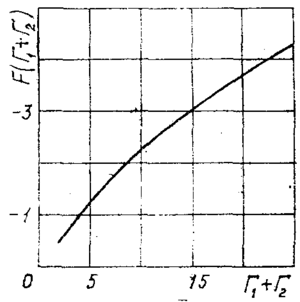

Fig. 3

pulse merger for a continuous series of values $\Gamma_1$ and $\Gamma_2$, we find the self-similar solution best approximating (3.2) in the limit of large $b\eta$ by following [1]. For a momentum I given in a first approximation, the self-similar solution $U^0$ of the BE is determined to the accuracy of a shift transformation of the independent variables $U^0 = U^0[(\xi + \xi^0)/(\eta + \eta^0)^{1/2}]$. Expanding $q^0$ in $\xi^0/(b\eta)^{1/2}$ and $(b\eta^0/b\eta)^{1/2}$, and q in x, we can obtain by a special selection of $\xi^0$ and $\eta^0$ that the amplitudes $q_m^0$ and $q_m$ will differ by terms of order $(b\eta)^{-2}$. As is easy to show, here

$$\omega^{-1}\xi^0 = 0,\ \omega^{-1}\eta^0 = \frac{\tau^2\omega}{4b}\frac{e^{\Gamma_1}(e^{\Gamma_1}-1)(e^{\Gamma_2}-1)}{(e^{\Gamma_1+\Gamma_2}-1)^2};$$

the desired self-similar solution corresponds to a short impact on the medium at the time $t = \omega^{-1}2\eta^0/(1+n)$, where the surface on which the impact is executed is to the left of the true boundary surface by a distance $m^0 = K_0\omega^{-1}2\eta^0/(1+n)$.

To estimate the merger distance, i.e., the emergence into the self-similar regime, we define this quantity as the distance in which $|q_m/q_m^0 - 1| \ll 1$. We have

$$b\eta_m \approx \theta\omega^2\tau^2\frac{[F(\Gamma_1+\Gamma_2)e^{\Gamma_1}(e^{\Gamma_1}-1)(e^{\Gamma_2}-1)(e^{\Gamma_1+\Gamma_2}-2e^{\Gamma_1}+1)]^{2/3}}{(e^{\Gamma_1+\Gamma_2}-1)^2},$$

$F(\Gamma_1+\Gamma_2)$ is the coordinate $y_m$ of the self-similar wave amplitude $q_m^0$ found as the solution of the equation

$$1+\frac{e^{\Gamma_1+\Gamma_2}-1}{\sqrt{\pi}}\left(\int_{-\infty}^{F} e^{-t^2}dt+\frac{1}{2}F^{-1}e^{-F^2}\right)=0.$$

The dependence $F(\Gamma_1+\Gamma_2)$ is shown in Fig. 3. Using the results of the numerical solution, we set the proportionality factor $\theta$ equal to $\theta = 9.2$ for any $\Gamma_1$ and $\Gamma_2 \leqslant 7$. Let us estimate the amplitude in distances (3.3)

$$q_m = -\frac{2b}{\sqrt{b(\eta_m+\eta^0)}}F(\Gamma_1+\Gamma_2). \tag{3.4}$$

Let us note that it is formally impossible to pass to the limit as $b \to 0$ in (3.3) since the expansions performed are valid only for large $b\eta$ and small $x < 1$. A comparison with the numerical solution shows that for $\Gamma_2 > 7$ [according to (3.3) this corresponds to $x_m > 1$] the merger distance $\eta_m$ diminishes more slowly than is given by (3.3). The qualitative features of the dependences of $x_m$ (or $\eta_m$) on $\Gamma_1$ and $\Gamma_2$, which are seen in Fig. 2, are described well by (3.3). Let us also note that the values of the amplitude $q_m$ computed by means of (3.3) and (3.4) for $\Gamma_2 \leqslant 7$ (it is necessary to set $\omega\tau \simeq 1$ here since $\tau_1 + \tau_2 \ll \tau$ by assumption) differ by not more than 5-10% from those which have been obtained because of the numerical solution.

Therefore, the selection of the delay time $\tau$ (if $\tau$ is selected as the characteristic duration $\omega^{-1} = \tau$, then we vary $\tau_1$ and $\tau_2$ in $\Gamma_1$ and $\Gamma_2$) permits obtaining a wave with given values of the amplitude, energy, and momentum at a given distance from the boundary on which the laser pulse train acts, say approximately in agreement with the same quantities for a wave corresponding to such a single action on the boundary $\eta = -\eta^0$ whose momentum in a first approximation will equal the momentum of the train. However, as already remarked, by using multiple action the losses because of shielding the surface of the condensed substance by rupture products are diminished substantially while a pulse of comparatively large duration is formed in the merger distance. In a number of cases taking account of the surface shielding influence turns out to be governing [2], and hence, the selection of the optimal shock formation conditions at a given distance from the boundary is associated with precisely the action of the pulse train on the substance boundary.

4. By knowing the characteristic distance of pulse merger and the change in amplitude during merger, the parameters of the resultant wave can be varied at a given distance from the boundary. The possibility hence exists for an increase in the total energy of the wave (as well as of other dynamic parameters) because of using the shock pulse interaction. It turns out that for a fixed impulse I the wave on the boundary (therefore for fixed work of an impactor on a substance) the selection of the lag time between the pulses permits raising the wave energy in a given distance. The reason for the effect is that varying the lag time permits minimizing the losses in the train, i.e., minimizing the entropy of the mass of substance between the boundaries and the surface on which it is required to increase the total energy, momentum, etc. This can be shown most simply in the case of large Re numbers when the solution of (1.1) with the arbitrary boundary condition $q(0, \xi) = \varepsilon f(\xi)$ has the form

$$q = \varepsilon f(\xi + \eta q). \tag{4.1}$$

As remarked above, the ambiguity arising in the profile q with the growth of $\eta$ is eliminated by using the "equal area rule" [6] from which in particular there follows

$$\int_{-\infty}^{+\infty} q\,d\xi = \varepsilon \int_{-\infty}^{+\infty} f(\xi)\,d\xi. \tag{4.2}$$

An important dependence between the amplitudes of the discontinuities in the wave profile and the quantity $E(\eta) = \int_{-\infty}^{+\infty} q^2 d\xi$ also results from the last equation and (4.1). To shorten the computations we assume that there is just one discontinuity, denoted by $R = \xi + \eta q$, $R_\pm = \xi_p + \eta q_\pm$, where $\xi_p(\eta)$ is the coordinate of the discontinuity, and $q_\pm(\eta)$ are the values of the function q behind and before the discontinuity, respectively. Then

$$\int_{-\infty}^{+\infty} q\,d\xi = \int_{-\infty}^{R_-} f(R)\left(1 - \eta f_R'\right) dR + \int_{R_+}^{+\infty} f(R)\left(1 - \eta f_R'\right) dR = \text{const}; \tag{4.3}$$

$$\int_{-\infty}^{+\infty} q^2 d\xi = \int_{-\infty}^{R_-} f^2(R)\left(1 - \eta f_R'\right) dR + \int_{R_+}^{+\infty} f^2(R)\left(1 - \eta f_R'\right) dR. \tag{4.4}$$

Evaluating the total derivative (4.4) with respect to $\eta$, we find by taking account of (4.3)

$$dE(\eta)/d\eta = -(1/6)(q_+ - q_-)^3. \tag{4.5}$$

It can be shown that in the case of several discontinuities there will be a sum of quantities $(q_-^{(i)} - q_+^{(i)})^3$ in the right side of (4.5). The equation (4.5) is a corollary of the energy conservation law for the case under consideration. In fact, the right side of (4.5) is proportional to the entropy jump on the discontinuity [1], while the left side with (4.3) taken into account is the change in the total energy of wave motion. The formula (4.5) rewritten in dimensional variables means that a change in the wave during passage from a Lagrange particle with coordinate m to an adjacent particle is associated with the change in enthalpy of the particle m. Formulas (4.5) can be obtained from the energy conservation law even for the initial system of hydrodynamic equations; however, we emphasize that nothing more than (4.1) and (4.2) is required for its derivation so that the result (4.5) is independent of the physical meaning of the initial system of equations.

Let us assume that a train of compression pulses of the form (2.1) is given on the boundary and (for simplification) $\varepsilon_1 = \varepsilon_2$, $\tau_1 = \tau_2$, $\omega^{-1} = \tau_1$, $\tau \ge 0$. Therefore, the impulse I on the boundary is fixed. Let us give a certain fixed value to the Lagrange coordinate $\bar\eta$, and let us examine how the quantity $\Delta = E(0) - E(\bar\eta)$ changes as a function of the lag time between pulses $\tau$. The dependence $\Delta(1 + \tau/\tau_1)$ is represented in Fig. 4 for $\mathrm{Re} \to \infty$, where $k = 1 + \tau/\tau_1$, $k_2 = (1 + \varepsilon_1 \bar\eta)^{1/2}$, $k_1 = k_2/(1 + \sqrt{2})$, $\lambda = 0.66(1 - 1/k_2)$, $\Delta_1 = 0.66(1 - \sqrt{2}/k_2)$. As is seen from Fig. 4, the minimum of $\Delta$ and the maximum of the wave energy hold for $1 \le k \le k_1$ $(k = k_1)$ and corresponds to merger of the discontinuities in $\bar\eta$. It hence follows from (4.5) that the entropy of the mass of substance between the boundary surface and $\bar\eta$ takes on a minimal value (for a given I). The fact that the mini-

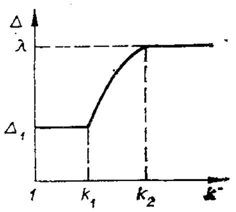


Fig. 4

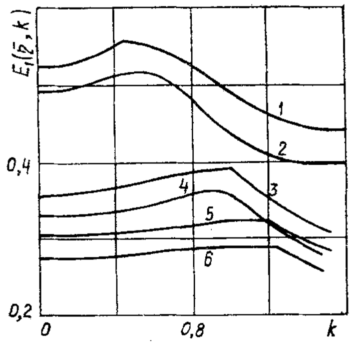


Fig. 5

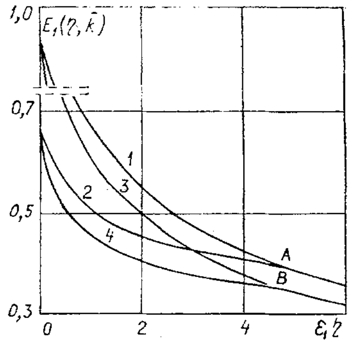


Fig. 6

mum entropy is achieved in a segment and not at a point is related to the fact that Fig. 4 is constructed for $\tau > 0$ (this question is discussed below). The domain $k \geq k_2$ in Fig. 4 corresponds to the case when the second pulse does not succeed in overtaking the first in $\eta \leq \bar{\eta}$, in this case both pulses are propagated independently.

Now, let us give conditions of type (2.1) on the boundary but such that $I_1 = \int_{-\infty}^{+\infty} u d\xi$ and $E_1 = \int_{-\infty}^{+\infty} u^2 d\xi$, where $u = q/\varepsilon_1$, remain constant in $\eta = 0$: $E_1(0) = E_0$, $I_1(0) = I_0$. We take the amplitude and duration of the leading pulse here as the characteristic scales of the amplitude and duration. Let us set $I_0 = 1$, $E_0 = 2/3$; these values correspond, for $k = 1$, say, to two successive triangular pulses with unit amplitudes, durations, and unit distance between discontinuities. We will then have on the boundary in place of (2.1)

$$u(0, \xi) = \begin{cases} 1 - \xi, & \xi \in [0, r), \\ (\mu + k - \xi)/\sigma, & \xi \in [k, k + \mu], \\ 0, & \xi \notin [0, r) \cup [k, k + \mu]; \end{cases} \tag{4.6}$$

k is the lag between discontinuities in the wave. For $k < 1$, $\bar{\mu} = [1 + (1 - k)^3]/[1 + (1 - k)^2]$, $\mu = [1 + (1 - k)^2]/\bar{\mu}$, $\sigma = \mu/\bar{\mu}$, $r = k$. For $k \geq 1$, $\bar{\mu} = 1$, $\mu = 1$, $\sigma = 1$, $r = 1$. It is seen that the conditions on the boundary are here functions of the one parameter k. If $I_1$ remains constant for $\eta > 0$, then $E_1$ characterizing the dynamic properties of the wave will diminish in conformity with (4.5). Now, for each fixed $\varepsilon_1\bar{\eta}$ we vary k on the boundary in order to obtain the greatest possible value of $E_1$ in $\bar{\eta}$. For the values $\mathrm{Re} \to \infty$ (nondissipative medium) and $\mathrm{Re} = 12$ the dependence $E_1(\bar{\eta}, k)$ is shown in Fig. 5. Curves 1 and 2 correspond to $\varepsilon_1\bar{\eta} = 1.2$; 3, 4) $\varepsilon_1\bar{\eta} = 4.82$; 5, 6) $\varepsilon_1\bar{\eta} = 7.4$, the first of each pair of curves corresponds to $\mathrm{Re} \to \infty$, the second to $\mathrm{Re} = 12$. For finite Re the problem (1.1), (4.6) was solved numerically. Let $\bar{k}$ be the value of the parameter k for which the maximum $E_1(\bar{\eta}, k)$ is achieved in $\bar{\eta}$. The dependences $E_1(\bar{\eta}, k)$ have distinctive form for $\bar{k} \leq 1$ and $\bar{k} > 1$. For $\bar{k} \leq 1$, the maximum $E_1(k)$ is related uniquely to the parameter k and corresponds to the boundary condition (4.6) for which merger of the discontinuities occurs in a given $\bar{\eta}$. For $\bar{k} > 1$ such uniqueness does not exist. If $k_* \geq 1$ governs the boundary condition (4.6), for which merger of the discontinuities occurs in $\bar{\eta}$, then the maximal $E_1(\bar{\eta}, \bar{k})$ is constant in the segment $\bar{k} \in [1, k_*]$. The curve 5 corresponds to $k_* = 1{,}2$ in Fig. 5. Therefore, if $\eta_0$ is the distance of discontinuity merger for $k_* = 1$ $[\eta_0 = 2(1 + \sqrt{2})/\varepsilon_1]$, then for fixed $\bar{\eta} > \eta_0$ the maximum $E_1$ can be obtained within the distance $\bar{\eta}$ and from the configuration $k_* = 1$ on the boundary $E_1(\bar{k} = 1, \bar{\eta} > \eta_0) = E_1(\bar{k} > 1, \bar{\eta})$.

Since the wave emerges on the asymptotic of a single pulse after merger of the discontinuities, then conditions corresponding to this single pulse can be given on the boundary $\eta = 0$

$$u(0, \xi) = \begin{cases} (k + \mu - \xi)/\sigma, & \xi \in [k + \mu - \sqrt{2\sigma}, k + \mu], \\ 0, & \xi \notin [k + \mu - \sqrt{2\sigma}, k + \mu]. \end{cases} \tag{4.7}$$

The solutions with conditions (4.7) and (4.6) [curves 1, 2 ($\mathrm{Re} \to \infty$) and 3, 4 ($\mathrm{Re} = 12$) in Fig. 6, respectively, for $k = 1$] agree with the point of wave discontinuity merger, while at this same point $\varepsilon_1\bar{\eta}$ the maximum $E_1(\bar{\eta}, k)$ is achieved. For $k_* \geq 1$ the configuration (4.7) does not change on the boundary, i.e., all the solutions $E_1(\eta, k_*)$ with the conditions (4.6) merge at the identical asymptotic (Fig. 6, curves 1 and 3 for $\eta > \eta_0$), by descending ever lower with the increase in $k_*$. Therefore, to obtain the maximal $E_1(\bar{\eta}, \bar{k})$ for $\bar{\eta} > \eta_0$, the configuration (4.6) with $k = 1$ must be given on the boundary.

The conditions on the boundary (4.6) and (4.7) have identical $I_1 = I_0$ but different $E_1$. Let $E_{01}$ correspond to (4.7). To obtain the identical value of the energy $E_1$ at the point of discontinuity merger, it is necessary to give $E_{01}/E_1 = \sqrt{2/\sigma}$ ($1 \leq \sigma \leq 2$, $\sigma = 1$ corresponds to $k = 1$) on the boundary. Therefore, waves with two discontinuities are energetically most favorable for distances from the boundary $\eta \geq \eta_0$ (where $\sigma = 1$) as compared with a solitary pulse (4.7) which yields a wave of the same profile in the merger distance of a train of two pulses. Let us note that the magnitude of the total energy for weak shock pulses in condensed media is a second order infinitesimal quantity. This is associated with the smallness of the initial pressure as compared to the compression modulus of the medium.

If a single pulse (4.7) with the total energy $\Sigma_1$ or a wave from two pulses with the total energy $\Sigma_2$ (4.6) is formed for $k = 1$ under action on the substance boundary, then to obtain an identical wave at the distances in a condensed medium, it is necessary to contribute a total energy

$$\frac{\Sigma_1}{\Sigma_2} = \frac{E_{01}}{E_0} = \sqrt{2} \tag{4.8}$$

times greater on the boundary in a single pulse. For media described by the equation of state of an ideal gas, this relationship has the form

$$\frac{\Sigma_1}{\Sigma_2} = 1 + \frac{(n-1)(\sqrt{2}-1)}{3/\varepsilon + n - 1}. \tag{4.9}$$

For $\varepsilon \leq 0.5$ formula (4.9) yields $\Sigma_1/\Sigma_2 \leq 1.1$ for $n = 3$. Formally here for $\varepsilon \to \infty$, $\Sigma_1/\Sigma_2 \to \sqrt{2}$.

The relationships (4.8) and (4.9) are valid for the case when dissipation can be neglected or when the Reynolds numbers take on large but finite values ($\mathrm{Re} \gg 1$).

As is seen from Fig. 5, the maximum $E_1(\overline{\eta}, k)$ in a dissipative medium is shifted somewhat relative to the maximum for no dissipation (for identical $\varepsilon_1\overline{\eta}$). Computations show that if the evolution of $E_1(\eta, \overline{k})$ is traced, then the distance $\varepsilon_1\eta$ at which the configuration (4.6) with $k = \overline{k}$ is optimal is achieved in a domain where $E''_{1\eta\eta}(\eta, \overline{k}) < 0$ (the domain B on curve 4 in Fig. 6), as $\mathrm{Re} \to \infty$ this domain degenerates into a point (the point A on curve 2 in Fig. 6), in which the first derivatives $E'_{1\eta}$ undergo a discontinuity, and the mean second derivative remains less than zero.

## LITERATURE CITED


1. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Academic Press.
2. S. I. Anisimov, Ya. A. Imas, G. S. Romanov, and Yu. V. Khodyko, Action of High Powered Radiation on a Metal [in Russian], Nauka, Moscow (1970).
3. R. Kinslaw, Hypervelocity Impact Phenomena [Russian translation], Mir, Moscow (1973).
4. É. I. Andriankin, "Crater formation in meteoric impact," Astron. Vestn., 12, No. 1 (1978).
5. É. I. Andriankin, "Collision of two plates at high velocity," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1963).
6. O. V. Rudenko and S. I. Soluyan, Theoretical Principles of Nonlinear Acoustics [in Russian], Nauka, Moscow (1975).
7. E. A. Dynin, "On compression pulse propagation in condensed media," Tr. VNIIFTRI, Physics of Impulsive Pressures [in Russian], No. 44(74) (1979).
8. A. A. Borisov, A. F. Vakhgel't, and V. E. Nakoryakov, "Propagation of longwave, finite-amplitude perturbations in gas suspensions," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1980).